ACTION OF PENDULUM IN TRANSIENT FLUID FLOW

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Abstract. The work is dedicated to the authors' latest research on the interaction of moving inflexible objects when subjected to non-constant velocity fluid flow (air, water) without the use of work-intensive space-time programming methods. In the first part of the study, the differential equation of the plane pendulum motion is derived using the novel approach of fluid-rigid body interaction phenomenon, in this equation, the moment caused by the fluid interaction is simplified by ignoring the flow viscosity. This makes it possible to obtain the usual second-order differential equation of pendulum motion, which contains components of relative velocity in a simplified way, instead of the partial differential equations in a continuous mathematical space. The application of the obtained equation is further used in solving specific tasks of engineering importance. The first task analyzes the pendulum swing motion in a still airflow. Here, the equation described above is numerically integrated and the results are compared with an experiment in a natural environment. The comparison resulted in a drag interaction factor that was further used in other more complex cases. The second task analyzes the pendulum motion when fluid flow velocities are a decreasing function of time in a harmonic behavior. In addition, in this case, the possibilities of applying the developed theory to other forms of flow rate change, such as pulse or poly harmonic forms, are considered. In the third task, the synthesis of motion control in a mechatronic system was performed. In this case, the possibility of regulating the additional resistance torque arising from the rotary damping generator is considered. The work is illustrated with graphical results. The outcomes obtained in the work can be used in the analysis of the interaction of existing moving objects with the fluid flow, as well as in the synthesis of new technological processes, for example, for obtaining energy from vibrating objects immersed in fluid flow.

Keywords: pendulum motion, space-time programming, fluid-rigid body interaction.

Introduction

Fluid-rigid body interaction phenomenon is studied to a great extent in the works [1-2], where an approximate analytical method by making use of the concept of zones is proposed. It is realized from the concept of zones that the space around the fluid rigid body interaction can be split into a pressure zone and a suction zone (or vacuum zone, where the pressure is constant along the length of the body). The main aim of the present work is to validate the approximated theory through the concept of zones and comparing with experiments in the real environment, as well as to predict the response of the pendulum in transient flow (flow parameters changing with time) by considering the phenomenon of rigid body-fluid interaction. The main aspects when dealing with the interaction phenomenon as analysed were the form of the rigid body, nature of flow (laminar or turbulent) and the determination of the interaction co-efficient (*C*) that is obtained through the concept of zones by analysing the interaction space. The interaction coefficient (*C*) can either be determined through computer numerical simulations or through experiments.

For validation for the interaction phenomenon, using a spherical bob pendulum, the proposed theoretical work involved two different ordinary differential equations to account for the dynamical behaviour of the pendulum immersed in water. The first equation considered the drag co-efficient and the next differential equation considered viscous damping co-efficient. Good agreement between the experimental and theoretical results was reported [3]. Dynamical behaviour of a non-linear model represented by motion of a damped spring pendulum immersed in inviscid fluid flow is investigated, using non-linear stability analysis, the effect of each parameter on the motion of pendulum is analysed and discussed [4]. An approximate mathematical model of a spring pendulum is formed to analyse the steady-state and transient motion of the system by varying the parameters. All analytical and numerical results were found to be in agreement [5]. A harmonically excited damped spring pendulum system with an attached rigid body is investigated utilizing the multiple scales technique that helped obtain asymptotic solutions to the governing equations of motion up to a good approximation. The accuracy of the multiple scales method was found to be good, and the time response of the solution is compared with the numerical results of the governing equations [6]. Fluid –structure interaction model by the dynamic mesh method using Fluent for a 2D mechanical heart valve simulation for a cardiac cycle was successfully validated by comparing the computer simulation results to experiments that are obtained from in vitro studies with the help of a CCD camera [7]. Experiments are specially designed for validation of numerical methods for an aero elasticity and fluid structure interaction problems for flexible filaments, which are of rectangular cross-section and different lengths in the wind tunnel. The structural response of the filaments was recorded. The results obtained showed that such experimental setup and approach is valuable for validation of numerical methods for aero elasticity [8]. A new CFD method based on block-iterative coupling for fluid structure interaction (Numerical aerodynamic simulation of NACA airfoil) is used. The numerical results were compared with the well-known experimental results. Further, airfoil flow simulations were performed to examine the several other influencing factors on aero elasticity and increased forced vibration amplitudes [9]. Numerically predicted wind forces and wind induced structural responses through numerical simulations are examined practically by considering aero elastic problems. The prediction of CFD technique for the response behaviour of the cylinder type structure and comparison with experiments was made to check for the accuracy and limitations of the technique [10].

Materials and methods

1. Model and method of thin plate interactions analysis

A model of thin plate (pendulum) in open air is shown in Figure 1. The z-axis of the pendulum rotation is horizontal, but the gravitational force is vertical and parallel to the y-axis. The pendulum has one degree of freedom (1DOF), which is the angle of rotation φ around the z axis. The pendulum has one degree of freedom (1DOF), which is the angle of rotation fi around the z axis.

During movement, the pendulum is subjected to support reactions, gravity and fluid resistance forces. In this work the rotational resistance about the axis is not considered, although it is very easy to do. The pendulum motion is described by the following integral-differential equation (1):

$$Jz \cdot \ddot{\varphi} = -m \cdot g \cdot \frac{L}{2} \cdot \sin(\varphi) - (1+C) \cdot B \cdot \rho \cdot \left[\int_0^L (\xi \cdot \dot{\varphi})^2 \cdot \xi \cdot d\xi \right] \cdot sign(\dot{\varphi}), \tag{1}$$

where Jz – moment inertia of pendulum, like $Jz = m \cdot L^2/3$;
$\ddot{\varphi}$, $\dot{\varphi}$, φ – angular acceleration, angular velocity and rotation angle;
g – free fall acceleration;
L, B – length and width of the plate;
C – constant, will be found in this work;
ρ – density of fluid;
ξ – radial coordinate of a small area with small length $d\xi$ (Fig. 1.).

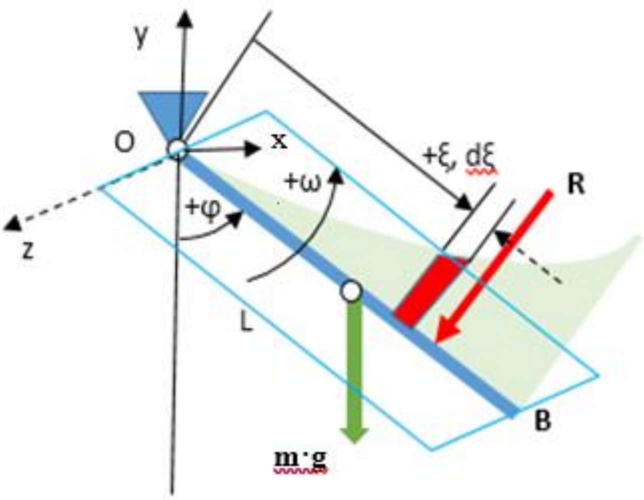

Fig. 1. **Free body diagram of the pendulum with fluid interaction (still flow)**

Since at the given time t the angular velocity $\dot{\varphi}$ is the same in all sections of the pendulum, the integral in equation (1) is simplified. Therefore, the equSation of the pendulum free decaying motion in the fluid flow will be as follows (2):

$$\ddot{\varphi} = -3 \cdot g \cdot \frac{1}{2} \cdot \sin(\varphi) - (1 + C) \cdot B \cdot \rho \cdot \left[\frac{3 \cdot L^2}{4 \cdot m} \cdot (\dot{\varphi})^2 \right] \cdot sign(\dot{\varphi}). \quad (2)$$

The graphs of the angles φ, φ_2 change (as a function of time t) in two cases (C = 0.5 and for C = 0.25) as shown in Fig. 2. One important conclusion can be drawn from the graphs: the period of damping motion is practically independent on the value of C, but the amplitudes of damping vibrations vary. This means that it is possible to determine this coefficient experimentally.

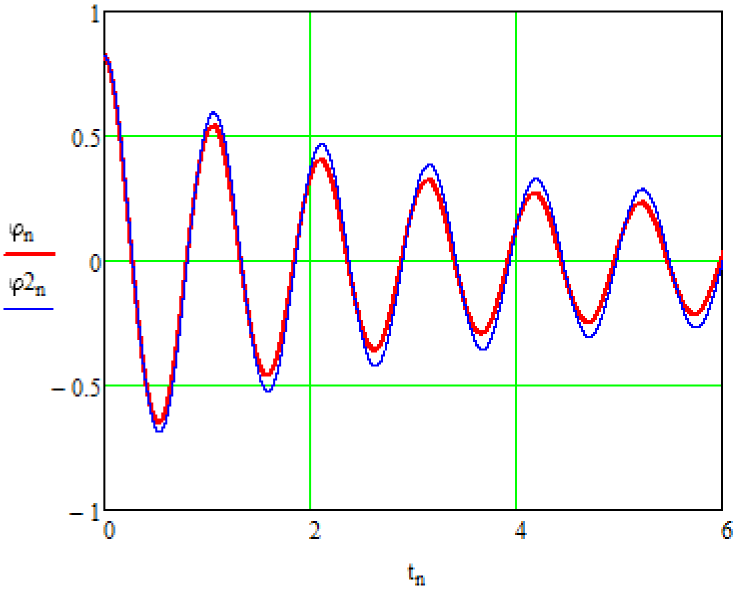

Fig. 2. **Dependence of damping oscillation angle amplitudes on the coefficient *C***

2. Experimental method of thin plate interactions analysis

Damping oscillations of a thin plate (pendulum) in open air or real environment is experimented that otherwise can be taken as an experimental setup to validate the interaction co-efficient C (2). The experimental setup consists of the thin flat plate sliding against the board marked with different angles as shown in Figure 3. The pendulum can be made to slide freely after it is hanged by the help of a horizontal support-rod placed at the top of the board. The oscillating pendulum motion can be seen in Figure 4.

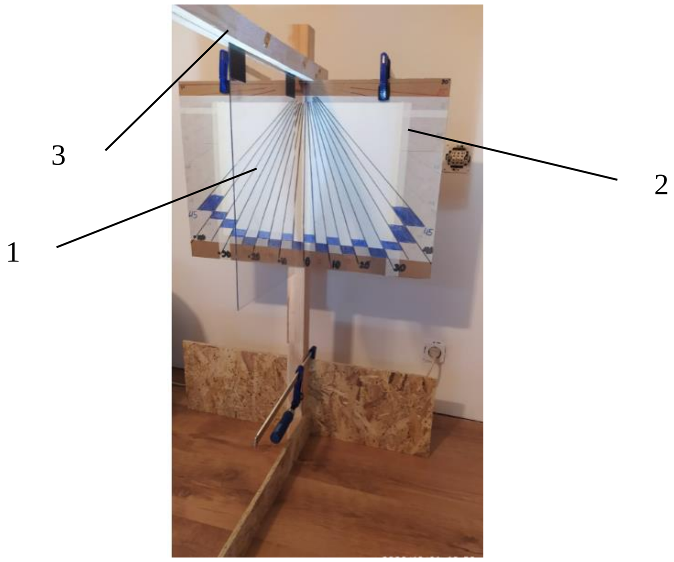

Fig. 3. **Experimental setup for pendulum oscillation motion:** 1 – freely hanging pendulum (thin plexiglas plate); 2 – pendulum sliding board with marked angles; 3 – horizontal support rod for pendulum

a) b)

Fig. 4. **Experimental setup for pendulum oscillation motion:**
a – from start position; b – from mean position

In the experiment setup a plexi-glass plate was attached to a horizontal beam with a thin adhesive tape. The plate was released for movement from a state of rest and at a certain initial angle (smaller deflections). Next, the experimental motion was recorded with a video camera, which was then compared with the motion according to equation (2). As it can be seen from equation (2), it has only one unknown value under given rules of motion, i.e., the coefficient *C*. By changing this coefficient *C*, a decreasing motion after a certain number of full oscillations was compared. For example, 5 oscillations were used in this experiment (Fig. 4.). The comparison of the experiment and theory at two different coefficients *C* is shown in Fig.5-6.

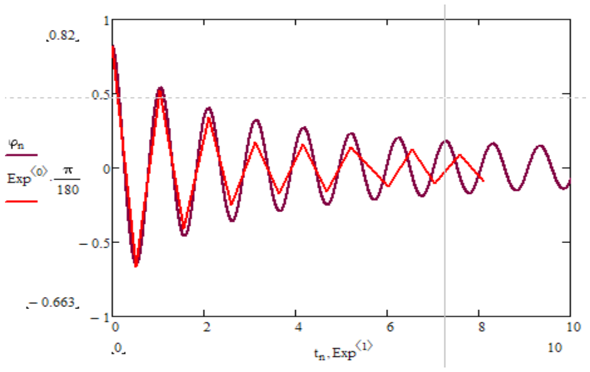

Fig. 5. **Angular displacement with time (comparison for experiment and numerical calculation) for *C* = 0.5**

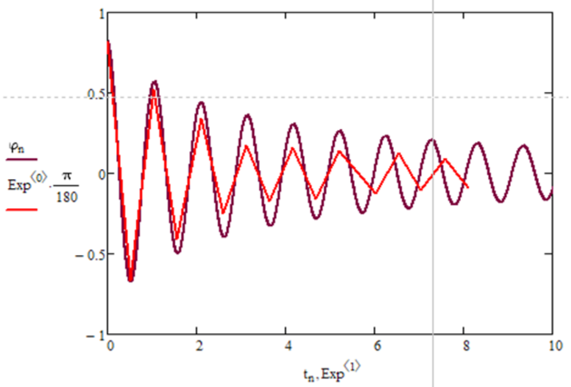

Fig. 6. **Angular displacement with time (comparison for experiment and numerical calculation) for *C* = 0.25**

It can be inferred from the graphs that the starting period from the numerical calculation was $T_s = 1.0774$ s and the starting period for experiments was $T_{se} = 1.02$ s, taking the value of the *C* coefficient of interaction as 0.5. With $C = 0.25$, the starting time period was $T_s = 1.0764$s. The overall percentage error was close to 5% in both the cases.

3. Non-stationary fluid flow interaction model

Consider the case when the fluid velocity V_0 is parallel to the x axis and does not change over time (Fig.1; 7). Consider the case when the fluid velocity V_0 is parallel to the x axis and does not change over time (Fig.1; 7).

To find the forces of fluid-pendulum interaction, find the relative velocity projections V_n on the pendulum normal plane.

It is as follows (3):

$$V_n = V_0 \cdot \cos(\varphi) - \xi \cdot \dot{\varphi}. \quad (3)$$

The pendulum integral – differential equation is simplified and has the following form (4):

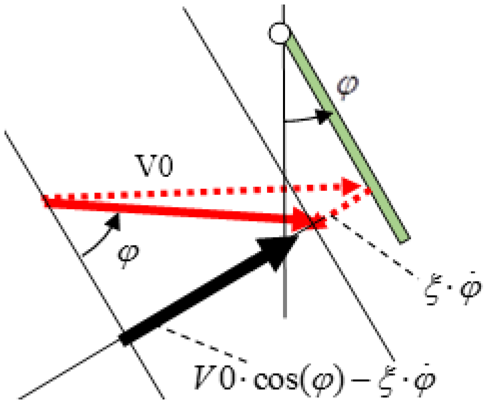

Fig. 7. **Relative velocity projections V_n to normal calculations**

$$\ddot{\varphi} = -3 \cdot g \cdot \frac{1}{2} \cdot \sin(\varphi) - (1 + C) \cdot B \cdot \rho \cdot \left[\frac{L^4 \cdot \dot{\varphi}^2}{4} - \frac{2 \cdot L^3 \cdot V_0 \cdot \dot{\varphi} \cdot \cos(\varphi)}{3} + \frac{L^2 \cdot V_0^{\,2} \cdot [\cos(\varphi)]^2}{2} \right] \cdot sign(V_0 \cdot \cos(\varphi) - L \cdot \dot{\varphi}). \quad (4)$$

The obtained equation (4) can be used to solve various problems of the pendulum motion when subjected to the fluid flow. The equation (4) could also help in analyzing the action of pendulum in transient or unsteady flow (changing flow velocity V_0).

It is also possible to synthesize motion by changing the pendulum area with an additional mechatronic system (area control action in case of perforated pendulum).

4. Mechatronic system analysisMechatronic system analysis

To illustrate the application of the developed theory, a pendulum system with a linear spring and a linear generator was considered (5):

$$\ddot{\varphi} = -3 \cdot g \cdot \frac{1}{2} \cdot \sin(\varphi) - (1 + C) \cdot B \cdot \rho \cdot \left[\frac{L^4 \cdot \dot{\varphi}^2}{4} - \frac{2 \cdot L^3 \cdot V_0 \cdot \dot{\varphi} \cdot \cos(\varphi)}{3} + \frac{L^2 \cdot V_0^{\,2} \cdot [\cos(\varphi)]^2}{2} \right] \cdot sign(V_0 \cdot \cos(\varphi) - L \cdot \dot{\varphi}) + \quad (5)$$

$$+ \{-c \cdot \varphi - b \cdot \dot{\varphi}\},$$

where c, b – constants.

The mathematical simulation results are illustrations for the flow harmonica velocity in Fig.8; 9. It should be noted that it is possible to calculate the energy obtained here using an appropriate (e.g., linear) mechatronic generator: *Power* = $b \cdot \dot{x}^2$, but the viscous nature of the fluid is ignored.

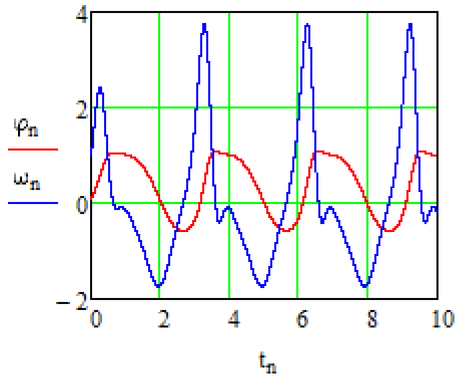

Fig. 8. **Angle and angular velocity as time function, when:** $V_0 = 10 \cdot (1 + \sin(1.12 \cdot t))$

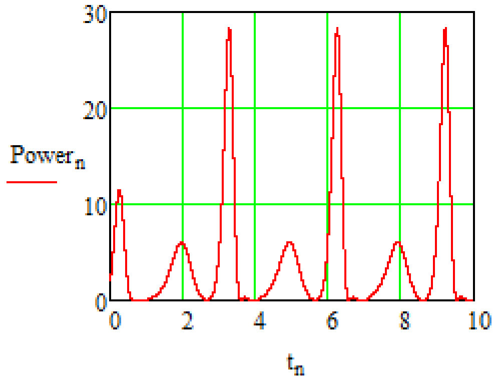

Fig. 9. **Power as time function:** *Power* = $b \cdot \dot{x}^2$

Results and discussion

The validation of the proposed theory [1-2] using the concept of zones through a simple pendulum setup was taken up in an air flow. The simple pendulum setup could be the best experimental device for quick validation of fluid structure interaction phenomenon as a similar type of validation with air, as fluid medium was performed for aero-elastic structures and NACA aero foils [8-9]. The motion of a plate pendulum can be described by an integral-differential equation, which, when integrated, transforms into a normal non-linear second-order differential equation. The integral differential equation could also consider the unsteady and steady nature of the fluid flow (changes in fluid velocity). In addition, it has been shown that an elastic spring element and a linear generator can be connected to the pendulum, which allows the pendulum to be used as an energy source.

Conclusions

1. The pendulum experimental setup could be the best simplest model to check for the fluid structure interaction phenomenon.
2. Through the principles of classical mechanics, the motion of the pendulum could be described using integral-differential equations when subjected to a fluid flow.
3. Ideas of energy extraction from fluid flow are offered if the system parameters can be changed with mechatronic systems.
4. An approximate method of fluid-structure interaction (ignoring viscous nature of the fluid medium) phenomenon is shown and used, which has its advantages in the synthesis of systems versus space-time numerical calculation methods.
5. The difference of starting periods for numerical simulations ($C = 0.5$ and $C = 0.25$) and experimental observations was found to be less than 5%.

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